PMM U.S.S.R.,Vol.46,pp.219-224 0021-8928/83/2 0219 $7.50/0

UDC 539.3:534.1

STABILITY OF A NONLINEARLY ELASTIC CYLINDER UNDER SIDE PRESSURE AND AXIAL COMPRESSION*

G.I. VOLOKITIN

There is considered the problem of loss of stability of thick-walled cylinders subjected to inhomogeneous initial stresses. The analysis is based on the exact three-dimensional equations of neutral equilibrium problems derived by superposing small on finite deformations. The law of the state is determined by a five-constant Murnaghan relationship. Cases are studied of the buckling of circular cylinders subjected to axial load and side internal and external hydrostatic pressure. Results are represented for calculations exhibiting the influence of nonlinearity on the upper critical load as a function of the geometric parameters.

There exists a comparatively small number of papers in which the stability of equilibrium is investigated on the basis of the three-dimensional equations of nonlinear elasticity theory /1/. Results are obtained for bodies of simple shape (slab, rod, hollow sphere, cylindrical tube), in which most solutions rely on a simplifying hypothesis regarding the incompressibility of the material or an assumption regarding the simple form of the state function. Mainly cases of neo-Hookean and semi-linear materials are studied /2,3/.

1. To investigate the stability of a nonlinearly elastic cylinder, we use the Lur'e neutral equilibrium equations /1/

$$\nabla\cdot\Theta = 0,\quad \Theta = T^{\cdot} + T\nabla\cdot\mathbf{w} - \nabla\mathbf{w}^T\cdot T \tag{1.1}$$
$$\mathbf{w} = \mathbf{R}^{\cdot} = \left[\frac{d}{d\eta}(\mathbf{R}+\eta\mathbf{w})\right]_{\eta=0},\quad T^{\cdot} = \left[\frac{d}{d\eta}T(\mathbf{R}+\eta\mathbf{w})\right]_{\eta=0}$$

Here ∇ is the nabla operator in the metric of the unperturbed state of strain, T is the Cauchy stress tensor, $\mathbf{R}$ is the radius-vector of a point of the body in the state of strain. By using the vector of additional displacements $\eta\mathbf{w}$ (η is a small parameter), the particle location in equilibrium modes adjacent to the subcritical is taken into account.

The five-constant Murnaghan dependence describes a broad class of isotropic nonlinearly elastic materials under moderate strains. For this state law the tensor Θ has the form /4/

$$\begin{aligned}\Theta = {} & I_3^{-1/2}\{{}^1/_8[-12\lambda - 8\mu + 9\nu_1 + 18\nu_2 + 8\nu_3 + (4\lambda - \\ & 6\nu_1 - 8\nu_2)I_1 + (\nu_1 + 2\nu_2)I_1^2 - (4\nu_2 + 8\nu_3)I_2]F\cdot\nabla\mathbf{w} + F\cdot\cdot \\ & \nabla\mathbf{w}\,[(\lambda - {}^3/_2\nu_1 - 2\nu_2)F + ({}^1/_2\nu_1 - 2\nu_3)I_1F + (\nu_2 + \\ & 2\nu_3)F^2] + (\nu_2 + 2\nu_3)F^2\cdot\cdot\nabla\mathbf{w}F + [\mu - {}^3/_2\nu_2 - 2\nu_3 - \\ & ({}^1/_2\nu_2 + \nu_3)I_1][F^2\cdot\nabla\mathbf{w} + F\cdot(\nabla\mathbf{w} + \nabla\mathbf{w}^T)\cdot F] + \\ & \nu_3I_3(2\nabla\mathbf{w}\cdot\cdot EE - \nabla\mathbf{w}^T)\}\end{aligned} \tag{1.2}$$

Here F is the Finger measure of strain, I_1, I_2, I_3 are its principal invariantes, λ, μ are Lamé coefficients, ν_1, ν_2, ν_3 are third order elasticity constants, E is the unit tensor, and the superscript T denotes the transpose.

If a load is applied to a body in the form of a uniformly distributed follower pressure, then the equilibrium equation for the additional strains is formulated on the surface in the form /1/

$$\mathbf{N}\cdot\Theta = -p(\nabla\cdot\mathbf{w}\mathbf{N} - \mathbf{N}\cdot\nabla\mathbf{w}^T) \tag{1.3}$$

Here p is the intensity of the pressure, $\mathbf{N}$ is the outer normal to the strained surface. Furthermore, this relationship is used in writing the boundary conditions on the outer and inner cylinder surfaces. Boundary conditions of a different kind are taken on the endfaces: for $z=0$ and $z=L$

$$\mathbf{i}_3\cdot\mathbf{w} = 0,\quad \mathbf{i}_3\cdot\Theta\cdot\mathbf{e}_r = \mathbf{i}_3\cdot\Theta\cdot\mathbf{e}_\varphi = 0 \tag{1.4}$$

*Prikl.Matem.Mekhan.,46,No.2,pp. 289-295,1982

The unit vector $\mathbf{i}_3$ is colinear to the axis of the unstrained cylinder whose length is L, while the inner and outer radii are r_1, r_0. The coordinate axis z is directed along the axis of the cylindrical shell. The conditions on the endfaces (1.4) specify no friction and inadmissibility of the additional displacements in the axial direction.

2. It is assumed that the subcritical strain of the cylinder is axisymmetric. By selecting the cylindrical (r, φ, z) coordinates which give the position of the point in the undeformed state as the material coordinates, we have for the coordinates of the deformed cylinder

$$R = R(r), \quad \varphi' = \varphi, \quad z' = \alpha z, \quad \alpha \text{ is const} \tag{2.1}$$

Such a deformed state occurs in a hollow circular cylinder under internal and external loading by hydrostatic pressure and compression from the endfaces by stiff smooth slabs. Condition (1.4) does not contradict such a loading.

The function $R(r)$ is determined as a result of solving the initial boundary value problem, the Lamé problem for a nonlinearly-elastic tube /1/

$$\sigma_r' + \frac{R'}{R}(\sigma_r - \sigma_\varphi) = 0, \quad \sigma_r(r_0) = -p_0, \ \sigma_r(r_1) = -p_1 \tag{2.2}$$

Here σ_r, σ_φ are the physical components of the stress tensor, and p_0, p_1 are the intensities of the external and internal pressures. The primes denote differentiation with respect to the coordinate r.

Expressions for the measure of the Finger strain and its principal invariants follow from (2.1)

$$\begin{gathered} F = \nabla\mathbf{R}^2 = a^2\mathbf{e}_r\mathbf{e}_r + b^2\mathbf{e}_\varphi\mathbf{e}_\varphi + \alpha^2\mathbf{i}_3\mathbf{i}_3 \\ I_1 = a^2 + b^2 + \alpha^2, \quad I_2 = a^2b^2 + \alpha^2(a^2 + b^2), \quad I_3 = a^2b^2\alpha^2 \\ (a(r) = 1 + u', \ b(r) = 1 + u/r) \end{gathered} \tag{2.3}$$

where $\mathbf{e}_r, \mathbf{e}_\varphi, \mathbf{i}_3$ are the basis vectors which coincide with the unit vectors of a cylindrical coordinate system, and the functions a, b are related to the radial displacement u by the equations presented above in parentheses. Using the known representation of the stress tensor in the Finger form, written with the Murnaghan relationship taken into account for the specific strain potential energy, we can give the components $\sigma_r, \sigma_\varphi, \sigma_z$ the form

$$\begin{aligned} \sigma_r &= \frac{2a}{b\alpha}\left[c^{(0)} + a^2c^{(1)} + \frac{c^{(-1)}}{a^2}\right] \\ \sigma_\varphi &= \frac{2b}{a\alpha}\left[c^{(0)} - b^2c^{(1)} + \frac{c^{(-1)}}{b^2}\right] \\ \sigma_z &= \frac{2\alpha}{ab}\left[c^{(0)} - \alpha^2c^{(1)} + \frac{c^{(-1)}}{\alpha^2}\right] \\ c^{(0)} &= 1/16\,[9\nu_1 + 18\nu_2 + 8\nu_3 - 4(3\lambda + 2\mu) + (4\lambda - \\ &\quad 6\nu_1 - 8\nu_2)I_1 + (\nu_1 + 2\nu_2)I_1^2 - (4\nu_2 + 8\nu_3)I_2] \\ c^{(1)} &= {}^1/_4\,[3\nu_2 + 4\nu_3 - 2\mu - (\nu_2 + 2\nu_3)I_1] \\ c^{-1} &= {}^1/_2\nu_3 I_3 \end{aligned} \tag{2.4}$$

The relationships (2.2) – (2.4) show that the boundary value problem determining the subcritical strain of the cylinder is nonlinear. In seeking its solution we must be constrained to approximate methods. For shells with small relative thickness the formula from linear theory is completely acceptable

$$u = \frac{p_1r_1^2 - p_0r_0^2}{2(\lambda + \mu)(r_0^2 - r_1^2)} + \frac{(p_0 - p_1)r_0^2r_1^2}{2\mu(r_0^2 - r_1^2)r} + \frac{(1 - \alpha)\lambda r}{2(\lambda + \mu)} \tag{2.5}$$

Such an approach, when the subcritical strains are determined by linear elasticity relationship in the formulation of the neutral equilibrium equations, has been developed sufficiently extensively. However, for thick-walled shells more exact expressions taking account of nonlinearity are of interest. In this case, a variation of the adjustment method described in /5/ was used to solve the initial boundary value problem (2.2) for cylinders with the relative thickness

$$\varepsilon = 2(r_0 - r_1)/(r_0 + r_1) > 0.05$$

It is convenient to take the solution of the Lamé problem for a cylinder of a semilinear material as the approximation in the first stage. The selection of the initial approximation by formulas from linear elasticity theory does not always assure convergence.

The cylinder loading in the axial direction is realized both because of the change in distance between the endfaces and because of the side pressure. The axial force Q is computed from the formula /1/

$$Q = 2\pi \int_{r_1}^{r_0} \sigma_z a R \, dr$$

Let us turn to an examination of the intermediate equilibrium conditions. We give the vector of the additional displacement in a form allowing nonsymmetric buckling modes

$$\mathbf{w} = u(r, \varphi, z)\,\mathbf{e}_r + v(r, \varphi, z)\,\mathbf{e}_\varphi + w(r, \varphi\, z)\,\mathbf{i}_3 \tag{2.6}$$

Substituting (1.2), (2.3), (2.6) into (1.1), and using the derivational formulas, we arrive at a system of differential equations describing the equilibrium in the bulk

$$\begin{aligned}
&\frac{\partial A_1}{\partial r} + \frac{a}{R}\left(A_1 - A_2 + \frac{\partial B_2}{\partial \varphi}\right) + \frac{a}{\alpha}\frac{\partial C_1}{\partial z} = 0\\
&\frac{\partial B_1}{\partial r} + \frac{a}{R}\left(B_1 + B_2 + \frac{\partial A_2}{\partial \varphi}\right) + \frac{a}{\alpha}\frac{\partial C_3}{\partial z} = 0\\
&\frac{\partial C_2}{\partial r} + \frac{a}{R}\left(C_2 + \frac{\partial C_4}{\partial \varphi}\right) + \frac{a}{\alpha}\frac{\partial A_3}{\partial z} = 0
\end{aligned} \tag{2.7}$$

where A_i, B_j, C_k $(i = 1, 2, 3; j = 1, 2; k = 1, 2, 3, 4)$ are linear combinations of partial derivatives of components of the vector w

$$\begin{aligned}
&A_i = A_{i1}\left(u + \frac{\partial v}{\partial \varphi}\right) + A_{i2}\frac{\partial u}{\partial r} + A_{i3}\frac{\partial w}{\partial z}\\
&B_j = B_{j1}\left(\frac{\partial u}{\partial \varphi} - v\right) + B_{j2}\frac{\partial v}{\partial r}\\
&C_k = C_{k1}\frac{\partial u}{\partial z} + C_{k2}\frac{\partial v}{\partial z} + C_{k3}\frac{\partial w}{\partial r} + C_{k4}\frac{\partial w}{\partial \varphi}\\
&A_{11} = \frac{a}{R}A_{22} = \frac{2a}{\alpha r}\xi(a, b, \alpha), \quad A_{12} = \frac{\zeta(a, b)}{\alpha b}, \quad A_{21} = \frac{\zeta(b, a)}{\alpha a r}\\
&A_{13} = \frac{a}{\alpha}A_{32} = \frac{2a}{b}\xi(a, \alpha, b), \quad A_{31} = \frac{\alpha}{R}A_{23} = \frac{2\alpha}{ar}\xi(b, \alpha, a)\\
&A_{33} = \frac{2}{ab}\left\{c^{(0)} + \left[2c_1 + 3c^{(1)} + \left(\frac{1}{2}\nu_1 - 2\nu_3\right)I_1\right]\alpha^2 + \right.\\
&\qquad \left.(2\nu_2 + 4\nu_3)\alpha^4\right\} + \nu_3 ab\\
&B_{11} = \frac{a}{R}B_{22} = \frac{a}{\alpha r}\left[c_4 + \left(\frac{1}{2}\nu_2 + \nu_3\right)I_1 - \nu_3\alpha^2\right]\\
&B_{12} = \frac{ar}{b}B_{21} = \frac{1}{\alpha b}\left[c_3 + c_6(a^2 + b^2) + c_7a^2b^2 + c_5(a^4 + b^4)\right]\\
&C_{11} = \frac{\alpha}{a}C_{23} = \psi(a), \quad C_{13} = \frac{\alpha}{a}C_{21} = \frac{\omega(b)}{b}\\
&C_{32} = \frac{\alpha r}{b}C_{44} = \psi(b), \quad C_{34} = \frac{\alpha}{br}C_{42} = \frac{\omega(a)}{ar}\\
&C_{12} = C_{14} = C_{22} = C_{24} = C_{31} = C_{33} = C_{41} = C_{43} = 0\\
&\xi(x, y, z) \equiv c_1 + c_0(x^2 + y^2) + {}^1/_4\,\nu_1 z^2\\
&\zeta(x, y) \equiv c_3 + 3c_6x^2 + c_2y^2 + {}^1/_2\,c_0y^4 + 3c_0x^2y^2 + 5\,c_5x^4\\
&\psi(x) \equiv \frac{2}{ab}\left[c^{(0)} + c^{(1)}(x^2 + \alpha^2)\right], \quad \omega(x) \equiv \alpha\left(2c^{(1)} - \nu_3x^2\right)\\
&c_0 = {}^1/_4\nu_1 + {}^1/_2\nu_2, \quad c_1 = {}^1/_2\,\lambda - {}^3/_4\,\nu_1 - \nu_2, \; c_2 = c_1 +\\
&\qquad {}^1/_4\,\nu_1\alpha^2\\
&c_3 = -{}^3/_2\,\lambda - \mu + {}^9/_8\,\nu_1 + {}^9/_4\,\nu_2 + \nu_3 + c_1\alpha^2 + {}^1/_2 c_0\alpha^4\\
&c_4 = \mu - {}^3/_2\,\nu_2 - 2\nu_3, \quad c_5 = {}^1/_8\,\nu_1 + {}^3/_4\,\nu_2 + \nu_3\\
&c_6 = c_2 + c_4 + {}^1/_2\,\nu_2\,\alpha^2, \quad c_7 = {}^1/_4\,\nu_1 + \nu_2 + \nu_3
\end{aligned}$$

The components of the additional displacement vector on the inner and outer cylinder surfaces are related by the conditions: for $r = r_1$ and $r = r_0$

$$A_1 = -p\left[\frac{1}{R}\left(u + \frac{\partial v}{\partial \varphi}\right) + \frac{1}{\alpha}\frac{\partial w}{\partial z}\right], \quad B_1 = \frac{p}{R}\left(\frac{\partial u}{\partial \varphi} - v\right), \quad C_2 = \frac{p}{\alpha}\frac{\partial u}{\partial z} \tag{2.8}$$

where $p = p_1$ if $r = r_1$, and $p = p_0$ if $r = r_0$.

We seek the solution of the system (2.7) and (2.8) in the form

$$\begin{aligned}
u &= X_{nm}(r)\cos n\varphi \cos \lambda_m z\\
v &= Y_{nm}(r)\sin n\varphi \cos \lambda_m z\\
w &= Z_{nm}(r)\cos n\varphi \sin \lambda_m z
\end{aligned}$$

Here n, λ_m are the wave-form parameters: λ_m is determined by $\lambda_m = \pi m/L$ in conformity with the boundary conditions on the endfaces (1.4), and the numbers n and m are integers. After separation of variables, the problem of bifurcation of the axisymmetric equilibrium of a nonlinearly elastic cylinder (2.7), (2.8) results in a homogeneous system of ordinary differential equations

$$\begin{aligned}
&A_{12}X''_{nm} + \left[A_{12}' + A_{11} + \frac{a}{R}(A_{12} - A_{22})\right]X'_{nm} + \\
&\quad n\left(A_{11} + \frac{a}{R}B_{22}\right)Y'_{nm} + \left[A_{11}' + \frac{a}{R}(A_{11} - A_{21} - n^2B_{21}) - \right.\\
&\quad \left.\lambda_m^2C_{23}\right]X_{nm} + n\left[A_{11}' + \frac{a}{R}(A_{11} - A_{21} - B_{21})\right]Y_{nm} + \\
&\quad (A_{13} + C_{21})\lambda_m Z'_{nm} + \lambda_m\left[A_{13}' + \frac{a}{R}(A_{13} - A_{23})\right]Z_{nm} = 0 \\
&B_{12}Y''_{nm} + \left[B_{12} - B_{11} + \frac{a}{R}(B_{12} + B_{22})\right]Y'_{nm} - \\
&\quad n\left(B_{11} + \frac{a}{R}A_{22}\right)X'_{nm} - \left[B_{11}' + \frac{a}{R}(B_{11} + B_{21} + n^2A_{21}) + \right.\\
&\quad \left.\frac{a}{\alpha}\lambda_m^2C_{32}\right]Y_{nm} - n\left[B_{11}' + \frac{a}{R}(B_{11} + B_{21} + A_{21})\right]X_{nm} - \\
&\quad n\lambda_m\left(\frac{a}{R}A_{23} + \frac{a}{\alpha}C_{31}\right)Z_{nm} = 0 \\
&C_{23}Z''_{nm} + \left(C_{23}' + \frac{a}{R}C_{23}\right)Z'_{nm} + \left(n^2\frac{a}{R}C_{44} + \lambda_m\frac{a}{\alpha}A_{33}\right)Z_{nm} + \\
&\quad \lambda_m\left[\left(C_{21} + \frac{a}{\alpha}A_{32}\right)X'_{nm} + \left(C_{21}' + \frac{a}{R}C_{21} + \frac{a}{\alpha}A_{31}\right)X_{nm} + \right.\\
&\quad \left. n\left(\frac{a}{R}C_{42} + \frac{a}{\alpha}A_{31}\right]Y_{nm}\right] = 0
\end{aligned} \tag{2.9}$$

and six boundary conditions: for $r = r_1$ and $r = r_0$

$$\begin{aligned}
&A_{12}X'_{nm} + \left(A_{11} + \frac{p}{R}\right)(X_{nm} + nY_{nm}) + \lambda_m\left(A_{13} + \frac{p}{\alpha}\right)Z_{nm} = 0 \\
&B_{12}Y'_{nm} - \left(B_{11} - \frac{p}{R}\right)(nX_{nm} + Y_{nm}) = 0 \\
&\lambda_m\left(\frac{p}{\alpha} - C_{21}\right)X_{nm} + C_{23}Z'_{nm} = 0
\end{aligned} \tag{2.10}$$

The critical quantities p_0, p_1 and α are determined by the eigenvalues of the problem (2.9), (2.10). In connection with the nonlinearity of the eigenvalue problem under consideration, numerical methods were used /4/. The algorithm to determine the bifurcation loads, including the solution of the boundary value problem of subcritical deformation and the evaluation of the eigenvalues of the system (2.9) and (2.10), is realized on an electronic computer.

3. Computations were executed in an example of a material whose elastic properties are described by the constants /4/: $\nu = 0.272$, $E = 2\cdot 10^{11}$ N/m^2, $\nu_1/E = -2.8$, $\nu_2/E = -2.1$, $\nu_3/E = -1$ (ν is the Poisson's ratio and E is Young's modulus). In order to clarify the influence of the nonlinearity the fundamental cases of loss of stability of circular cylinders, studied in shell theory /6/, were examined.

For very long shells under external pressure, the critical load was sought in the form $p_0 = p_* E\varepsilon^3$. Numerical analysis showed that wave formation with $n = 2$, $m = 0$ (plane buckling modes) correspond to the critical (minimal) value of the parameter p_*. For thin-walled shells ($\varepsilon < 0.01$) the p_* agrees with the classical value determined by the Grashoff-Bresse formula /6/. Critical values of $p_*\cdot 10^3$ are compared below for the following versions: 1) $\nu_1 = \nu_2 = \nu_3 = 0$, the initial problem is linear; 2) $\nu_1 = \nu_2 = \nu_3 = 0$, the initial problem is nonlinear; 3) $\nu_1 = -2.8E$, $\nu_2 = -2.1E$, $\nu_3 = -E$, the initial problem is linear, 4) the constants ν_1, ν_2, ν_3 are nonzero and the initial problem is nonlinear

ε	0.0122	0.05	0.1	0.2	0.333	0.5
1)	270	259	246	217	175	144
2)	270	269	267	264	242	226
3)	270	260	248	225	172	126
4)	270	270	277	296	318	408

Additional axial compression results in a certain increase in the critical values of the parameter p_*.

For shells of medium length, the critical load was sought in the form $p_0 = p_* E\varepsilon^{2.5}$ in conformity with the Southwell—Papkovich formula. Let us present the values of $p_* \cdot 10^3$ corresponding to the case $\alpha = 1$, $\nu_1 = \nu_2 = \nu_3 = 0$, $r_0 / L = 0.382$, $m = 1$

ε	0.002	0.005	0.01	0.04	0.08	0.2	0.333
n	8	6	5	4	3	3	2
$p_* \cdot 10^3$	347	358	360	375	386	440	447
$p_{*l} \cdot 10^3$	348	358	362	395	427	452	502

The last row contains the value of $p_* \cdot 10^3$ for the variant in which the displacements of the subcritical state are found by linear theory. Taking account of the third order constants ν_1, ν_2, ν_3 results in a certain increase in p_* (by 10 - 15% for thick-walled cylinders). The addition of an axial load contributes to a reduction in the critical pressure.

Table 1

	Long cylinders m=1, n=1			Medium length cylinders m=1, n=0			Short cylinders m=1, n=0		
ε	0.0122	0.1	0.333	0.0122	0.1	0.333	0.0122	0.1	0.333
λ_m	0.08	0.2	0.4	16.53	5.774	4.082	30	10	6
a)	997 3.2	980 20	918 68	993 7.3	938 59	— —	987 12.6	917 90	700 142
b)	997 3.2	986 16	933 23	993 7.3	944 52	809 134	986 12.6	912 73	641 162
c)	996 3.3	985 18	926 29	993 7.3	964 47	730 340	986 12.6	867 191	569 509

Table 1 yields a representation of the influence of the physical and geometric nonlinearities on the magnitude of the upper critical force for the longitudinal compression for long, medium, and short cylinders. The pairs of numbers in the cells of this table are the critical values of the elongation parameter $\alpha \cdot 10^3$ (first row) and the axial compressive stress $\sigma \cdot 10^3$ (second row). Data are presented for the cases: a) initial displacements are found by the formula (2.5), the third order constants are not taken into account; b) the initial displacements are determined more exactly from (2.1)—(2.3), where $\nu_1 = \nu_2 = \nu_3 = 0$; c) the third order constants are not zero, and the initial problem is nonlinear. In all the cases considered here the effect due to the addition of external pressure is identical, the critical compression force is raised.

As a last example we present the results for a cylinder loaded from within and from outside by identical pressure ($p_0 = p_1$). The cylinder length remains unchanged ($\alpha = 1$). By verifying the possibility of buckling in three-dimensional modes due to side pressure only, we seek the critical load in the form $p_0 = p_1 = p_* E\varepsilon$. Considering comparatively thin-walled cylinders, we omit the moduli ν_1, ν_2, ν_3, and find the initial displacements by linear theory:

Table 2

ε λ_m	0.0122 16.53	0.0244 11.688	0.0488 8.265
$n=0$	4.26	3.99	4.01
$n=1$	4.43	4.06	4.10
$n=2$	4.80	4.60	6.27

Table 2 contains the bifurcation values of the parameter p_* (the axial stresses are computed by the formula $|\sigma_{z,p}| = 2\nu p_* E\varepsilon$). It is seen that the wave formation over the length corresponds to critical forces of medium length shells under longitudinal compression. These forces are themselves determined by the Lorentz—Timoshenko formula

$$\sigma_z = [3(1-\nu^2)]^{-1/2} E\varepsilon \approx 0.6 E\varepsilon$$

By analyzing the results of the computation, the deduction can be made that $|\sigma_{z,p}| > \sigma_z$.

The results presented below for the case $\varepsilon = 0.0122$, $n = 0$ show that numbers n, m exist for shells of given length, to which the least load will correspond as follows

λ_m	21	10	7	6	5	3	1
p_*	5.67	3.09	3.06	2.99	3.10	3.17	3.21

Values of p_* for $n=1$ and $n=2$ are omitted here, which exceed the value of p_* for $n=0$. The example shows that a nonlinearly elastic cylinder between fixed slabs can become unstable due to a follower hydrostatic pressure acting on the inner and outer side surface. This result characterizes the non-trivial distinction between compressible and incompressible nonlinearly elastic solids. Such cases of loss of stability are impossible in the latter circumstances /7/.

The author is grateful to L.M. Zubov for attention to the research.

REFERENCES

1. LUR'E A.I., Nonlinear Theory of Elasticity. NAUKA, Moscow, 1980.
2. SENSENIG C., Instability of thick elastic solids, Communs. Pure and Appl. Math., Vol.17, No.4, 1964.
3. HAUGHTON D.M. and OGDEN R.W., Bifurcation of inflated circular cylinders of elastic material under axial loading. II. Exact theory for thick-walled tubes. J. Mech. Phys. Solids, Vol. 27, No.5,6, 1979.
4. VOLOKITIN G.I., Influence of physical and geometrical nonlinearity on the value of the upper critical pressure in the buckling of a hollow sphere, PMM, Vol.42, No.3, 1978.
5. SHAMANSKII V.E., Methods of Numerical Solution of Boundary Value Problems on an Electronic Digital Computer. Pt.2. NAUKOVA DUMKA, Kiev, 1966.
6. GRIGOLIUK E.I. and KABANOV V.V., Shell Stability. NAUKA, Moscow, 1978.
7. ZUBOV L.M. On uniqueness conditions in the small for the state of hydrostatic compression of a solid, PMM, Vol.44, No.3, 1980.

Translated by M.D.F.